A COMPARISON OF THE METHODS OF COMPUTING THE WIND STRESSES IN BUILDINGS

A THESIS

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BY

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OF ADVANCED DEGREES

INTRODUCTION

The tall building frame of today constitutes one of the most indeterminate structures in common use when the wind stresses are considered. This fact is not generally recognized because the very tall building in which the wind stresses are of major importance is of comparative recent origin and because the maximum forces (wind storms) for which they must be designed are of rare occurance. Therefore, a structure inadequately designed for wind stresses may serve for many years without giving any indication of weakness.

The uncertainties involved in the proper design of wind bracing involve the following:

1. Wind velocities to be provided against and the resulting wind pressures and their distribution over the exposed surface. In other words, the determination of a proper equivalent uniform wind pressure for buildings of varying heights and widths.
2. Methods of calculation, which involve the validity of the assumptions made.
3. The working or unit stresses to be used in proportioning the members of the structure.

Even after assumptions have been made with regard to the above uncertainties, the calculation of the exact stresses in a many storied, many panelled frame without diagonals is so laborious as to be impractical. Dr.C.A.Melick attempted to devise a method for the exact determination of these stresses, but found the process so long that when applied to a building only four stories high, the amount of work required was almost prohibitive. To apply it to a building fifteen or twenty stories high would be impractical, if not, in fact, impossible. Other men also have attempted to develop accurate methods of determining the stresses in buildings due to the wind but they too have been so long as to be impractical. In general practice wind stresses are calculated by "approximate methods". There are a number of approximate methods of analysis in common use, in all of which assumptions are made with regard to the location of the points of contraflexure in the members and the distribution of the shears or direct stresses among the columns. Examples of the most used of these methods will be worked out later on in this paper.

The definitions of the most important terms used in this thesis are given below.

kip - a contraction of "kilo - pound" meaning 1000 pounds

bent - a vertical section passed through the building including the columns and the girders in the plane of the section

anemometer - a device for measuring the wind's velocity

point of contraflexure - the point in a member where the bending moment equals zero

WIND PRESSURE

The first thing the engineer must decide, in determining the wind stress a building frame must resist, is the maximum wind pressure likely to occur. The pressure exerted by the wind on a structure depends upon its velocity; hence, the prime importance of measuring the velocity of the wind correctly. During the past two centuries all kinds of devices have been used to measure the velocity of the wind. The velocities obtained by all methods are more or less in error, some of them to a very great extent. The Robinson Four-cup Anemometer, or some modification of it, has been used perhaps more than all other instruments combined. The two great merits of this instrument are its simplicity and the absence of a weather vane. However, it leaves a short but violent gust of wind unrecorded. After several experiments Dr. Robinson deducted that no matter what the size of the cups or the length of the arms, "the centers of the hemispheres move with one-third of the wind's velocity, except so far as they are retarded by friction". Later this deduction was proven to be incorrect.

The U.S.Weather Bureau prescribes that each pattern of anemometer should have its particular law of rotation determined by special experiment. Its standard instruments (where not replaced by the three-cup anemometer) have hemispherical cups four inches in diameter on arms of a length 6.72 inches from the vertical axis to the center of the cups. In order to obtain the true velocity a correction formula is applied to the indicated velocity. The correction formula recommended by Professor Marvin and adopted by the Weather Bureau is

$$\text{Log } V = 0.509 + 0.9012 \log v$$

in which V is the true or actual velocity of the wind and v is the indicated linear velocity of the cup centers, both in miles per hour. As an illustration consider an indicated velocity of 60 miles per hour. This means that the velocity of the center of the cups is

$$1/3 \times 60 = 20 \text{ m.p.h.}$$

the one-third relation being incorporated in the train of gears which counts the number of revolutions made by the anemometer cups. To compute the true velocity by the formula by logarithms we have:

(1) log v = log 20 = 1.30103

(2) log (log 20) = 0.11428

(3) log 0.9012 = 9.95482-10

(4) sum (2) and (3) = 0.06910

antilog 0.06910 = 1.1725

add 0.5090

log V = 1.6815

V = 48.03

The true velocity can also be obtained from the indicated velocity by means of Table I. These values are from actual observations of several instruments in a wind tunnel and may differ in some cases slightly from the values obtained by the Marvin formula. This table was compiled by Messrs.

S.P.Fergusson and R.N.Covert and appeared in "Monthly Weather Review", April 1924.

Beginning on January 1, 1928 the Weather Bureau put into use the new three-cup anemometers at all first-order stations in continental United States. These three-cup anemometers were developed after much study and experiment and have proven more satisfactory than the four-cup anemometers. The velocities indicated by this instrument are so close to the true velocities of the wind that errors in the anemometer itself are smaller than errors from other sources, such as those due to exposure, variability in velocity during the time period chosen, the mechanical condition of the anemometer, and limitations in making and interpreting the record. The error does not reach 1 per cent until a velocity of 30 miles per hour is reached and does not reach 4 per cent until 80 miles per hour is reached. Therefore, the indicated values are recorded, reported and published without correction.

The velocity of the wind increases with height because the friction of the air and the earth reduces velocity near the ground. It is therefore important to know at what height the velocity was recorded and also whether or not the anemometer was sheltered by nearby buildings or other obstacles.

The maximum wind velocity to be resisted having been determined the next step is to calculate the pressure this velocity will produce. Sir Isaac Newton was perhaps the first to find the relation between velocity and pressure. He found that the pressure p varies as dv^2, where d is the density and v the velocity. The density of the air being constant, the law for wind pressure is that the pressure varies directly as the square of the velocity; which has remained almost undisputed since Newton's time. Newton also said that for an area of unity $p = dh$, in which $h(=\frac{v^2}{2g})$ is the height through which a heavy body must fall to acquire the velocity v, g being the acceleration due to gravity.

Marburg in his "Framed Structures and Girders" says:

"Theoretically the pressure p, in pounds per square foot, on a plane surface normal to the direction of flow of a fluid having a relative velocity v, in feet per second, is equal to the weight of a vertical column of the fluid, having a cross-section of 1 square foot and a height h, in feet, equal to that through which a freely moving body must fall to acquire the velocity v. If w denotes the weight of the fluid, in pounds per cubic foot,

$$p = wh = \frac{wv^2}{2g}$$

For air at a temperature of 32° F. and at a barometric pressure of 760 mm., $w = 0.081$. Letting $g = 32.2$

$$p = 0.00126\ v^2$$

If V denotes the velocity of the wind in miles per hour, $v = 1.47\ V$, whence

$$p = 0.0027\ V^2$$

Expressing this equation in the general form

$$p = c\ V^2$$

in which c is an empiric coefficient. Experimental investigations of wind pressure on thin plates indicate that the actual value of this coefficient is greater than 0.0027. Its value is found to be somewhat affected by the velocity of the wind, and by the size and shape of the plate, but chiefly by the formation of a partial vacuum at the back of the plate which tends to increase greatly the resultant pressure in the direction of the wind".

William Ferrel, who deduces the relation between pressure and velocity after the theory of Newton, says:

"What is usually called the force of the wind upon any given object, as a plate of a given area, is the difference of the pressure on the two sides. On the one side the pressure is increased, not only by the momentum of the air, but likewise by the dragging effect through friction of the air which passes by, upon the cone or pyramid of comparatively stationary air in front of the plate or barrier. On the other side the effect, to the same amount, is to drag the air away and to diminish the pressure of the air. Hence, the theoretical difference of pressure, or effective force of the wind, is increased by equal amounts by the effect of friction upon the pressure of both sides. Accordingly, the force of wind determined experimentally is found to be a little greater than the theoretical pressure given by the formula".

Professor Marvin, now chief of the U.S. Weather Bureau but at that time in charge of the Instrument Division, as the result of extended observations on Mount Washington concluded that the formula for computing wind pressure under a barometric pressure of 30 inches may be best expressed by

$$p = 0.004 \, V^2$$

in which p is the intensity of pressure in pounds per square foot and V is the true velocity in miles per hour. Professor Marvin's recommendation for the value of c as well as his correction formula for V were adopted by the Weather Bureau and have been widely used.

Wind pressure has also been measured direct and independent of the velocity. The most favorable method of measuring wind pressure direct is by the Dines pressure-tube anemometer. This instrument is a modification of the Pitot tube and is especially adopted to recording pressure during gusts and rapid velocity changes. A vertical tube is bent horizontally at tha top and the open end is kept facing the wind, this tube being connected to the inside of a hollow float, which is made to rise by an increase of pressure. The vertical tube passes inside of a larger vertical tube having perforations, and as the wind blows across these perforations, it sucks out of the larger tube which at the bottom is connected with a chamber above the float, so that the suction causes the float to rise just as pressure in the other tube causes it to rise.

The pressure plate anemometer, originally invented by Sir Hiram S. Maxim, in 1909, has a pressure plate 13 inches in diameter, kept directed against the wind by a vane, the motion of the plate indicating the pressure or the velocity by moving an arm around a graduated scale. Another instrument is a pendulum which is blown from the vertical position by the wind, the divergence indicating velocity or pressure.

Direct measurments of wind pressure are seldom made. When wind pressures are reported, they are generally obtained from measurements of velocity, and it is important to know how they were made, what formula was used, and whether the recorded or the true velocity was used. Most anemometers indicate too high a velocity.

Most cities specify in their building codes the minimum wind pressure to be resisted. These codes vary greatly in their requirements. Some specify from 15 to 30 pounds per square foot to be taken over the entire exposed surface while others may require from 10 to 35 pounds per square foot over parts.

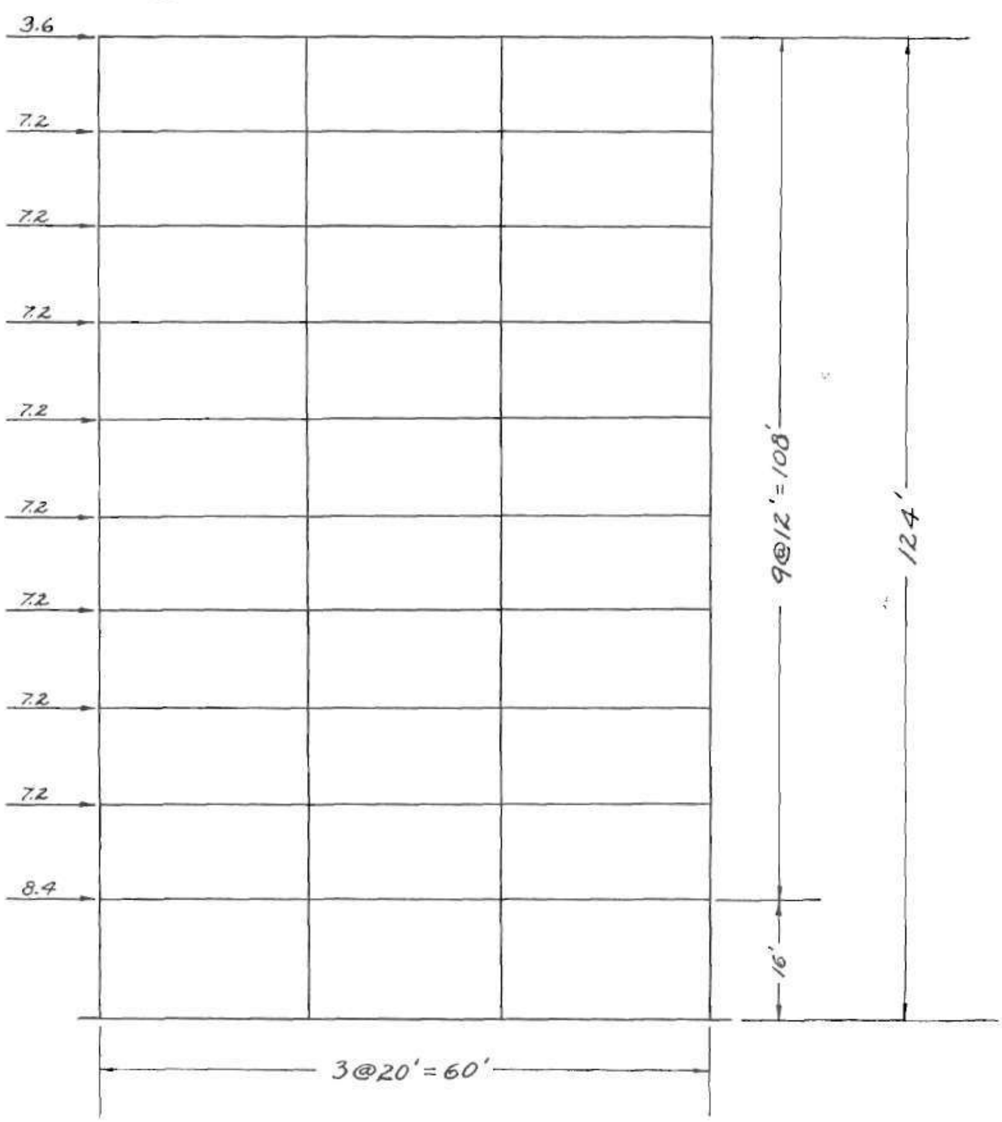
TABLE I

Correct or true velocities corresponding to velocities indicated by the Standard Four-cup Robinson Anemometer of the Weather Bureau (in miles per hour).

VELOCITY	
INDICATED	CORRECT
5	5.1
10	9.3
15	13.5
20	17.3
25	21.3
30	24.9
35	28.7
40	32.3
45	36.0
50	39.7
55	43.4
60	47.0
65	50.7
70	54.4
75	58.0
80	61.7
85	65.3
90	69.1
95	72.9
100	76.5

METHODS OF COMPUTING WIND STRESSES

The various approximate methods will be applied to a bent from a ten story building having three equal aisles. A wind pressure of 30 pounds per square foot is assumed acting over the entire exposed surface. The bents are considered as being spaced 20 feet on centers. A sketch of the bent is shown below. The wind loads are in kips.



American Insurance Union Building, Columbus, Ohio
Wind stresses determined by Cantilever Method

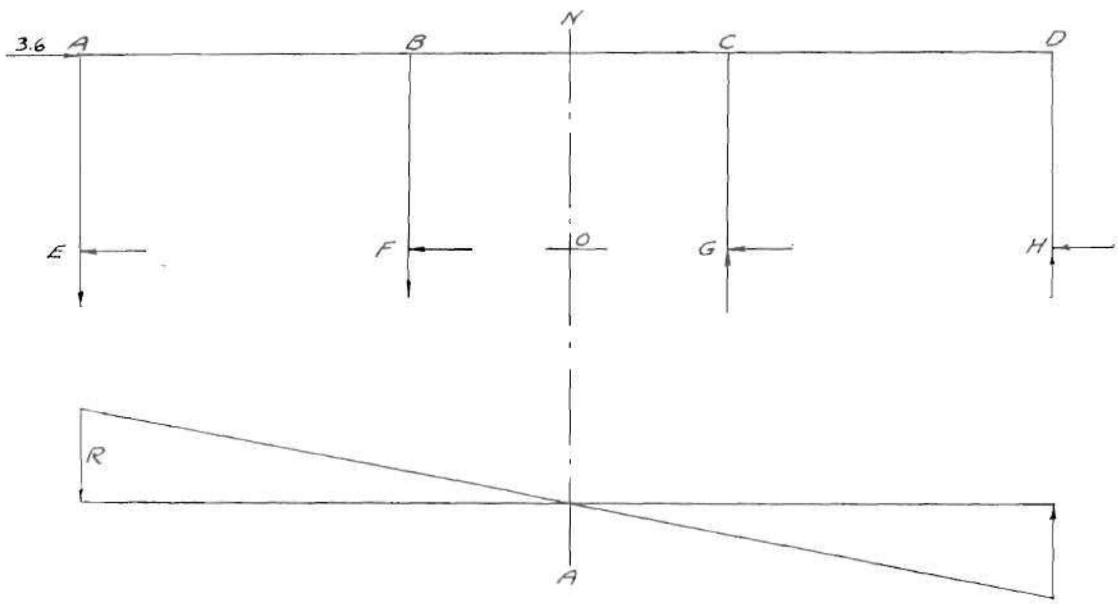
THE CANTILEVER METHOD

For convenience this method is called the Cantilever Method, although in all methods the structure as far as wind forces are concerned is a cantilever. This method is based upon the following assumptions:

1. A bent of a frame acts as a cantilever.
2. The point of contraflexure of each column is at mid-height of the story.
3. The point of contraflexure of each girder is at its mid-length.
4. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.
5. The wind load is resisted entirely by the steel frame.
6. It is usually futher assumed that all columns in a story have the same sectional area.

As the aisles are the same width, the neutral axis of the bent coincides with the center line of the building. The wind loads are applied horizontally at the intersection of the columns and girders. The windward columns receive direct tensile stress and the leeward columns receive direct compressive stress. Since the direct stress in a column is directly proportional to its distance from the neutral axis of the bent, the stresses in the interior columns are equal to each other and to one-third the stresses in the exterior columns. The columns and the girders connecting to them will be considered fixed. Any part of the structure between points of contraflexure can be considered as an independent structure.

Considering that part of the structure above the points of contraflexure of the columns in the tenth story we have the independent structure shown below.



The vertical reactions on the columns will vary directly as their distance out from the neutral axis as shown above. For the moment of the wind load about point O we have

$$M = 3.6 \times 6 = 21.6 \text{ kip-ft.}$$

This moment must be balanced by the moment of the vertical reactions on the columns. Let R be the vertical reaction at E. Then the vertical reaction at a point one foot out from point O would be (R/30), and so the vertical reaction at F and G will be (R/30)10 and at E and H it will be (R/30)30. Taking moments about O we have

$$2\left[(R/30)30^2 + (R/30)10^2\right] = 21.6$$

from which we get

$$66.67\ R = 21.6$$

$$R = 0.32 \text{ kip}$$

The number 66.67 is constant for all the stories.

The vertical reaction at F and G will be

$$\frac{0.32}{30} \times 10 = 0.11 \text{ kip}$$

The vertical reaction at H will of course be equal to that at E, 0.32 kip.

With the vertical reactions on the columns known we can determine the vertical shear on the girders by simply adding up the vertical forces. Beginning at A we have for the shear in girder AB

$$V = 0.32 \text{ kip}$$

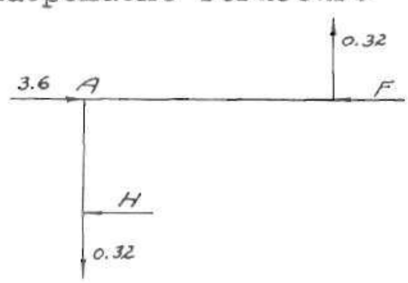
For the shear in girder BC we have

$$V = 0.32 + 0.11 = 0.43 \text{ kip}$$

For the shear in girder CD we have

$$V = 0.32 + 0.11 - 0.11 = 0.32 \text{ kip}$$

Having the vertical reactions on the columns and the vertical shears on the girders determined, we can calculate the horizontal shears on the columns. Considering that part of the structure shown below as an independent structure and taking moments about A we have



$$6\ H = 0.32 \times 10$$

$$H = 0.54 \text{ kip}$$

The moment in the column is

$$M = 0.54 \times 6 = 3.24 \text{ kip-ft.}$$

The moment in the girder is

$$M = 0.32 \times 10 = 3.20 \text{ kip-ft.}$$

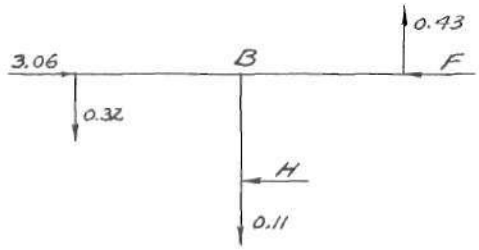
The summation of forces along the horizontal is

$$3.60 - 0.54 - F = 0$$

$$F = 3.06 \text{ kips}$$

for the direct compressive stress in girder AB.

Considering that part of the structure shown below as an independent structure and taking moments about B we have



$$6 H = 0.32 \times 10 + 0.43 \times 10$$

$$H = 1.26 \text{ kips}$$

The moment in the column is

$$M = 1.26 \times 6 = 7.56 \text{ kip-ft.}$$

The moment in the girder is

$$M = 0.43 \times 10 = 4.30 \text{ kip-ft.}$$

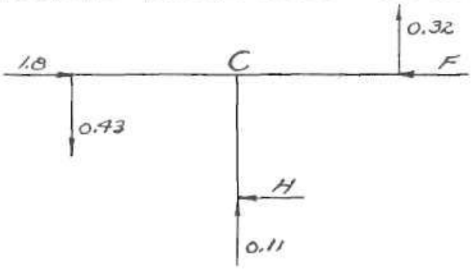
The summation of forces along the horizontal is

$$3.06 - 1.26 - F = 0$$

$$F = 1.80 \text{ kips}$$

for the direct compressive stress in girder BC.

Considering that part of the structure shown below as an independent structure and taking moments about C we have



$$6 H = 0.43 \times 10 + 0.32 \times 10$$

$$H = 1.26 \text{ kips}$$

The moment in the column is

$$M = 1.26 \times 6 = 7.56 \text{ kip-ft.}$$

The moment in the girder CD is

$$M = 0.32 \times 10 = 3.20 \text{ kip-ft.}$$

The summation of forces along the horizontal is

$$1.80 - 1.26 - F = 0$$

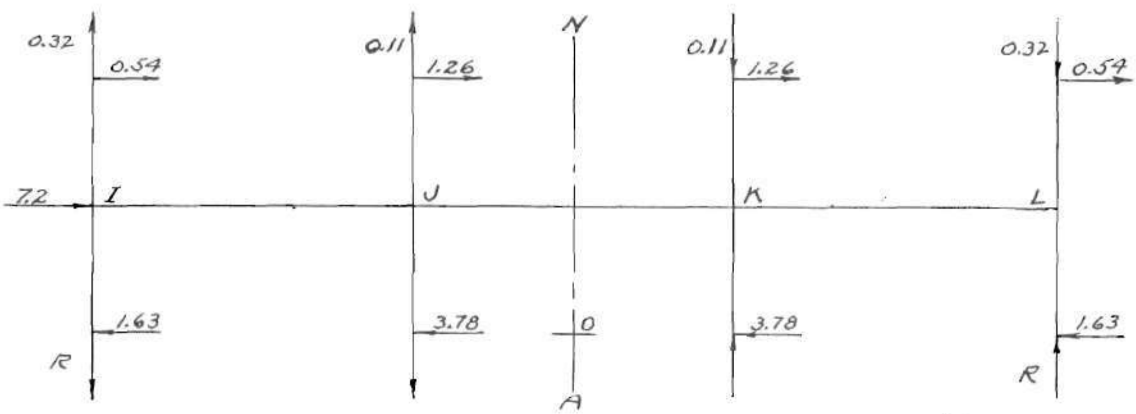
$$F = 0.54 \text{ kip}$$

for the direct compressive stress in girder CD.

The horizontal shear in column D must equal the direct stress in girder CD which is 0.54 kip.

It can be seen that the moments and shears are symmetrical about the center line of the bent so that it is only necessary to compute the moments and shears in the members of one half of the bent.

Next take as an independent structure that part of the structure between the points of contraflexure in the columns of the tenth and ninth stories as shown below.



Taking moments about point 0 of the wind forces acting on the building above this point we have

$$M = 3.6 \times 18 + 7.2 \times 6 = 108 \text{ kip-ft.}$$

This moment must be balanced by the moment of the vertical reactions on the columns. If R is the vertical reaction on an exterior column of the ninth story we have

$$R = \frac{108}{66.67} = 1.62 \text{ kips}$$

Then the vertical reaction on an interior column will be

$$\frac{10}{30} \times 1.62 = 0.54 \text{ kip.}$$

The vertical shears on the girders are found by beginning at the left and summing up the vertical forces acting on the independent structure. For the shear in girder IJ we have

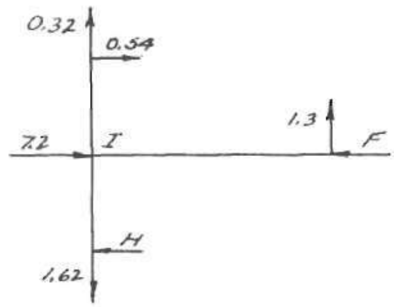
$$V = 1.62 - 0.32 = 1.30 \text{ kips.}$$

For the shear in girder JK we have

$$V = 1.62 - 0.32 + 0.54 - 0.11 = 1.73 \text{ kips.}$$

The shear in girder KL is the same as that in girder IJ, which is 1.30 kips.

Considering that part of the structure shown below as an independent structure and taking moments about I we have



$$6\ H = 1.30 \times 10 - 0.54 \times 6$$

$$H = 1.63 \text{ kips.}$$

The moment in the column is

$$M = 1.63 \times 6 = 9.78 \text{ kip-ft.}$$

The moment in the girder IJ is

$$M = 1.30 \times 10 = 13.00 \text{ kip-ft.}$$

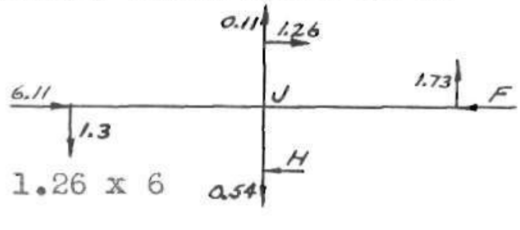
The summation of forces along the horizontal is

$$7.20 + 0.54 - 1.63 - F = 0$$

$$F = 6.11 \text{ kips}$$

for the direct compressive stress in girder IJ.

Considering that part of the structure shown below as an independent structure and taking moments about J we have



$$6\ H = 1.73 \times 10 + 1.30 \times 10 - 1.26 \times 6$$

$$H = 3.78 \text{ kips.}$$

The moment in the column is

$$M = 3.78 \times 6 = 22.68 \text{ kip-ft.}$$

The moment in girder JK is

$$M = 1.73 \times 10 = 17.30 \text{ kip-ft.}$$

The summation of forces along the horizontal is

$$6.11 + 1.26 - 3.78 - F = 0$$

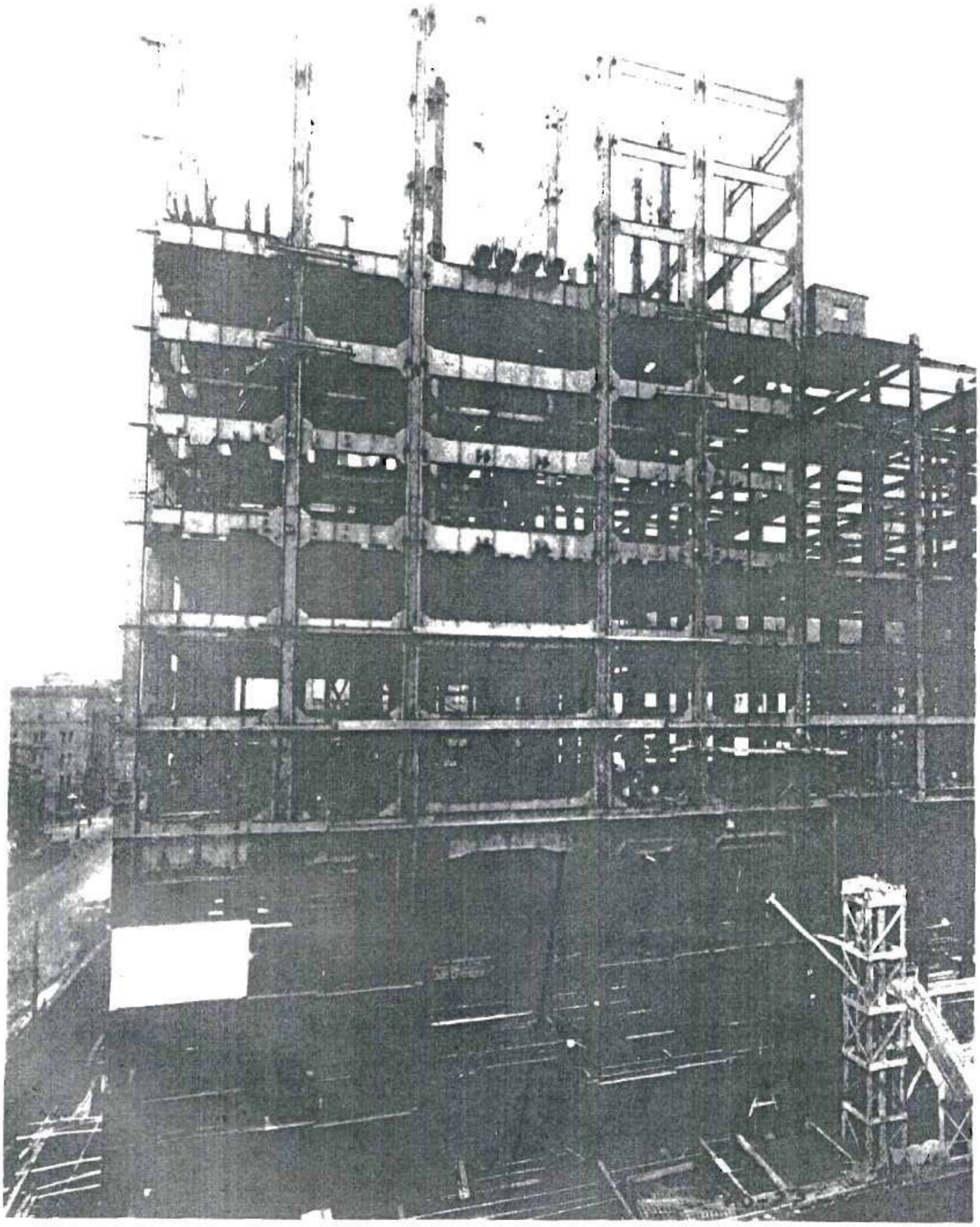
$$F = 3.59 \text{ kips.}$$

Since the bent is symmetrical about the center line the shears and moments of the members of the right half will be the same as tne shears and moments of the members of the left half.

Continuing in the same manner as shown above the shears and moments of the members of the other stories are determined. It can be seen from the tabulated results that the horizontal shear on each column varies from the ninth story down to the second story by a constant and that the vertical shear on the girders in each aisle varies by a constant from the tenth floor down to the third floor. Hence, after the shears in the members of the three top stories have been calculated, these constants can be determined and used to find the shears and moments in the remaining columns and girders. After the vertical shears in the girders have been found, the direct stresses in the columns can be determined by beginning at the ninth story and adding up the vertical forces included between points of contraflexure.

V=0.32 M=3.20 H=0.54 M=3.24 T=0.32	V=0.43 M=4.30 H=1.26 M=7.56 T=0.11	
V=1.30 M=13.0 H=1.63 M=9.78 T=1.62	V=1.73 M=17.3 H=3.78 M=22.68 T=0.54	
V=2.59 M=25.9 H=2.70 M=16.20 T=4.21	V=3.46 M=34.6 H=6.30 M=37.80 T=1.40	
V=3.89 M=38.9 H=3.78 M=2[illegible].68 T=8.10	V=5.18 M=51.8 H=8.82 M=52.92 T=2.70	
V=5.18 M=51.8 H=4.86 M=29.16 T=13.28	V=6.91 M=69.1 H=11.34 M=68.04 T=4.43	
V=6.48 M=64.8 H=5.94 M=35.64 T=19.76	V=8.64 M=86.4 H=13.86 M=83.16 T=6.59	
V=7.78 M=77.8 H=7.02 M=42.12 T=27.54	V=10.37 M=103.7 H=16.38 M=98.28 T=9.18	
V=9.07 M=90.7 H=8.10 M=48.60 T=36.61	V=12.10 M=121.0 H=18.90 M=113.4 T=12.20	
V=10.37 M=103.7 H=9.18 M=55.08 T=46.98	V=13.82 M=138.2 H=21.42 M=128.52 T=15.66	
V=13.86 M=138.6 H=10.44 M=83.52 T=60.84	V=18.48 M=184.8 H=24.36 M=194.88 T=20.28	

THE CANTILEVER METHOD



Steel work of the lower eight stories of
American Insurance Union Building

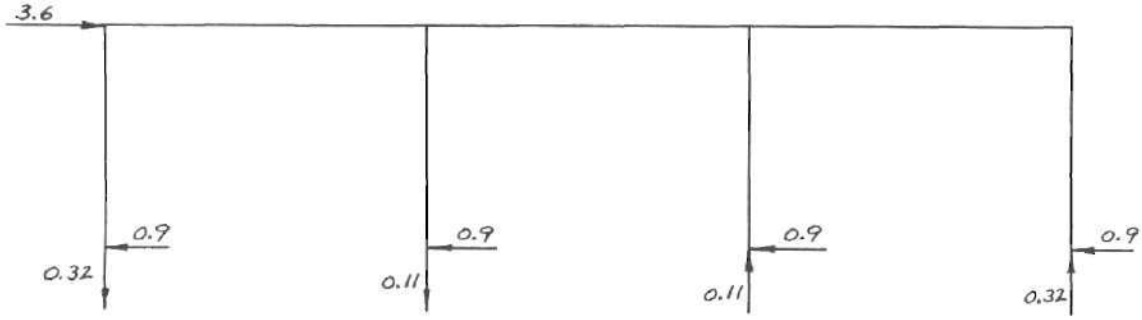
THE CONTINUOUS PORTAL METHOD

The Continuous Portal Method is based upon the following assumptions:

1. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.
2. The horizontal shear on any plane is equally distributed among the columns cut by that plane.
3. The sectional areas of all columns on a story are the same.
4. The point of contraflexure of each column is at mid-height of the story.

The direct stresses in the columns are found in the same way and are equal in amount to the column stresses calculated by the Cantilever Method. The vertical shears in the girders are also the same as in the Cantilever Method.

Considering that part of the structure above the points of contraflexure of the tenth story columns we have the independent structure shown below.



Since the horizontal shear is divided equally among the columns, each of the tenth story columns will take 0.90 kip. The moment in a tenth story column will be

$$M = 0.90 \times 6 = 5.40 \text{ kip-ft.}$$

Since the vertical shears in the girders are the same as in the Cantilever Method, the moments in the girders will also be the same.

In a like manner the shears and moments of the members of the other stories are calculated.

V=0.32 M=3.20 H=0.90 M=5.40 T=0.32	V=0.43 M=4.30 H=0.90 M=5.40 T=0.11	
V=1.30 M=13.00 H=2.70 M=16.2 T=1.62	V=1.73 M=17.30 H=2.70 M=16.2 T=0.54	
V=2.59 M=25.9 H=4.50 M=27.0 T=4.21	V=3.46 M=34.6 H=4.50 M=27.0 T=1.40	
V=3.89 M=38.9 H=6.30 M=37.8 T=8.10	V=5.18 M=51.8 H=6.30 M=37.8 T=2.70	
V=5.18 M=51.8 H=8.10 M=48.6 T=13.28	V=6.91 M=69.1 H=8.10 M=48.6 T=4.43	
V=6.48 M=64.8 H=9.90 M=59.4 T=19.76	V=8.64 M=86.4 H=9.90 M=59.4 T=6.59	
V=7.78 M=77.8 H=11.70 M=70.20 T=27.54	V=10.37 M=103.7 H=11.70 M=70.20 T=9.18	
V=9.07 M=90.7 H=13.5 M=81.0 T=36.61	V=12.10 M=121.0 H=13.5 M=81.0 T=12.2	
V=10.37 M=103.7 H=15.3 M=91.8 T=46.98	V=13.82 M=138.2 H=15.3 M=91.8 T=15.66	
V=13.86 M=138.6 H=17.40 M=139.2 T=60.84	V=18.48 M=184.8 H=17.40 M=139.2 T=20.28	

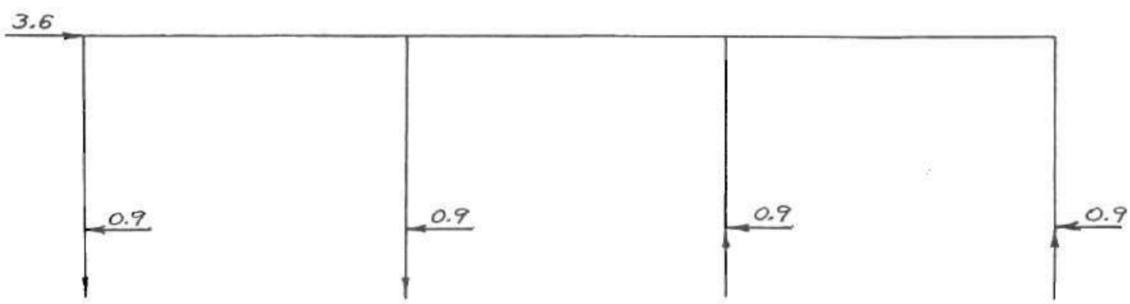
THE CONTINUOUS PORTAL METHOD

THE METHOD OF EQUAL SHEARS

The Method of Equal Shears is based upon the following assumptions:

1. The horizontal shear on any plane is equally distributed among the columns cut by that plane.
2. The point of contraflexure of each column is at mid-height of the story.
3. The wind load is resisted entirely by the steel frame.

Considering that part of the structure above the points of contraflexure of the tenth story we have the independent structure shown below.



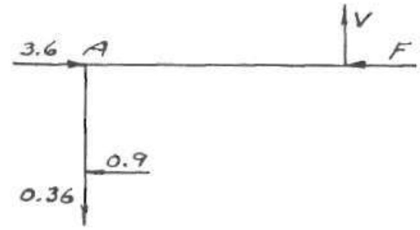
The horizontal shear produced by the wind is distributed equally among the four columns, giving each column a shear of 0.90 kip. The direct stresses coming upon any interior column from adjacent aisles are equal in amount but opposite in direction; therefore, their algebraic sum is zero and only the exterior columns have direct stresses.

Taking moments about the point of countraflexure of one of the exterior columns of the tenth story we have for the direct stress in the other exterior column

$$60\ S = 3.60 \times 6$$

$$S = 0.36 \text{ kip}$$

Considering that part of the structure shown below as an independent structure and taking the summation of forces along the horizontal we have



$$3.60 - 0.90 - F = 0$$

$$F = 2.70 \text{ kips}$$

for the direct stress in girder AB.

The summation of forces along the vertical is

$$V = 0.36 \text{ kip}$$

for the shear in girder AB.

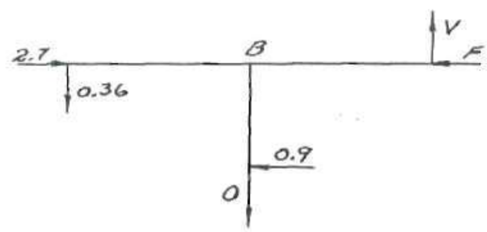
The moment in girder AB is

$$M = 0.36 \times 10 = 3.60 \text{ kip-ft.}$$

The moment in the column is

$$M = 0.90 \times 6 = 5.40 \text{ kip-ft.}$$

Considering that part of the structure shown below as an independent structure and taking the summation of forces along the horizontal we have



$$2.70 - 0.90 - F = 0$$

$$F = 1.80 \text{ kips}$$

The summation of forces along the vertical is

$$V = 0.36 \text{ kip}$$

for the shear in girder BC.

The moment in girder BC is

$$M = 0.36 \times 10 = 3.60 \text{ kip-ft.}$$

The moment in the column is

$$M = 0.90 \times 6 = 5.40 \text{ kip-ft.}$$

As the bent is symmetrical about the center line the shears and moments of the members of the other half will be the same as those just calculated. The other stories are analized in the same manner.

V=0.36 M=3.60 H=0.90 M=5.40 T=0.36	V=0.36 M=3.60 H=0.90 M=5.40 T=0.00	
V=1.44 M=14.4 H=2.70 M=16.2 T=1.80	V=1.44 M=14.4 H=2.70 M=16.2	
V=2.88 M=28.8 H=4.50 M=27.0 T=4.68	V=2.88 M=28.8 H=4.50 M=27.0	
V=4.32 M=43.2 H=6.30 M=37.8 T=9.00	V=4.32 M=43.2 H=6.30 M=37.8	
V=5.76 M=57.6 H=8.10 M=48.6 T=14.76	V=5.76 M=57.6 H=8.10 M=48.6	
V=7.20 M=72.0 H=9.90 M=59.4 T=21.96	V=7.20 M=72.0 H=9.90 M=59.4	
V=8.64 M=86.4 H=11.7 M=70.2 T=30.6	V=8.64 M=86.4 H=11.7 M=70.2	
V=10.08 M=100.8 H=13.5 M=81.0 T=40.68	V=10.08 M=100.8 H=13.5 M=81.0	
V=11.52 M=115.2 H=15.3 M=91.8 T=52.2	V=11.52 M=115.2 H=15.3 M=91.8	
V=13.08 M=130.8 H=17.4 M=139.2 T=65.28	V=13.08 M=130.8 H=17.4 M=139.2	

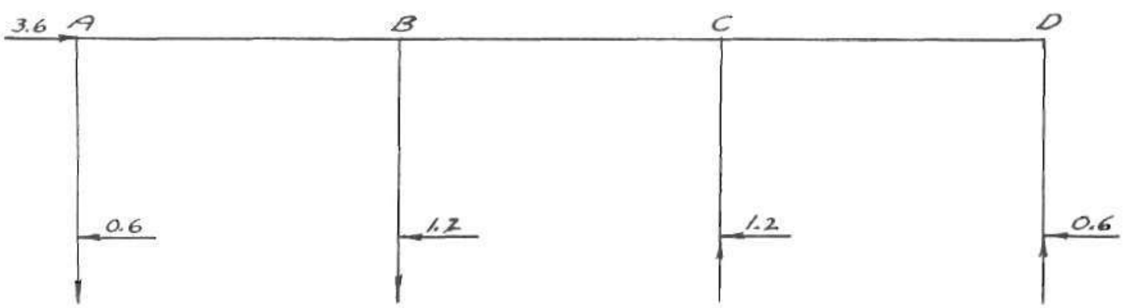
THE METHOD OF EQUAL SHEARS

THE PORTAL METHOD

The Portal Method is a special case of the Method of Equal Shears and is based upon the following assumptions:

1. A bent of a frame acts as a series of independent portals.
2. The point of contraflexure of each column is at mid-height of the story.
3. The point of contraflexure of each girder is at its mid-length.
4. The horizontal shear on any plane is divided equally between the aisles. An outer column thus takes only one-half as much shear as an interior column.
5. It is usually futher assumed that all columns in a story have the same sectional area, otherwise the horizontal shear in the columns is proportional to the moments of inertia of the columns.

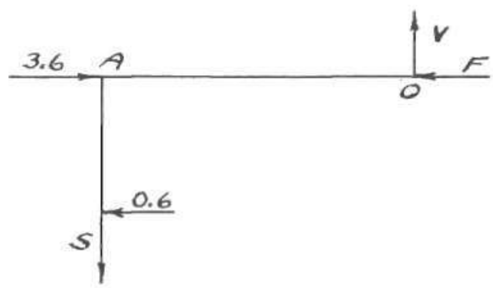
Considering that part of the structure above the points of contraflexure of the tenth story columns we have the independent structure shown below.



The shear produced by the wind load is divided equally between the three aisles, which gives 1.20 kips per aisle.

Each exterior column will take one-half of the shear of an exterior aisle, or one-half of 1.20 kips which is 0.60 kip. The interior columns must each take one-half of the shear of an exterior aisle and one-half of the shear of an interior aisle, or 1.20 kips each.

Considering that part of the structure shown below as an independent structure and taking moments about 0 we have



$$10\ S = 0.60 \times 6$$

$$S = 0.36 \text{ kip}$$

for the tension in the column.

The summation of forces along the vertical is

$$V = 0.36 \text{ kip}$$

for the vertical shear in the girder.

The moment in the girder is

$$M = 0.36 \times 10 = 3.60 \text{ kip-ft.}$$

The summation of forces along the horizontal is

$$3.60 - 0.60 - F = 0$$

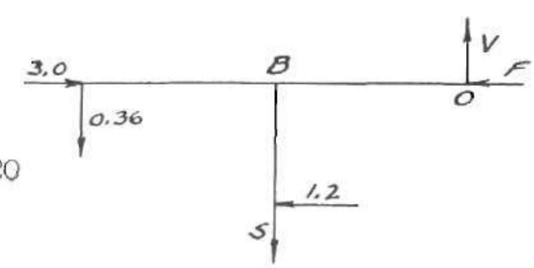
$$F = 3.00 \text{ kip}$$

for the direct compression in the girder.

The moment in the column is

$$M = 0.60 \times 6 = 3.60 \text{ kip-ft.}$$

Considering that part of the structure shown below as an independent structure and taking moments about 0 we have



$$10\ S = 1.20 \times 6 - 0.36 \times 20$$

$$S = 0$$

for the stress in the column.

The moment in the column is

$$M = 1.20 \times 6 = 7.20 \text{ kip-ft.}$$

The summation of forces along the vertical is

$$V = 0.36 \text{ kip}$$

for the vertical shear in the girder.

The moment in the girder is

$$M = 0.36 \times 10 = 3.60 \text{ kip-ft.}$$

The summation of forces along the horizontal is

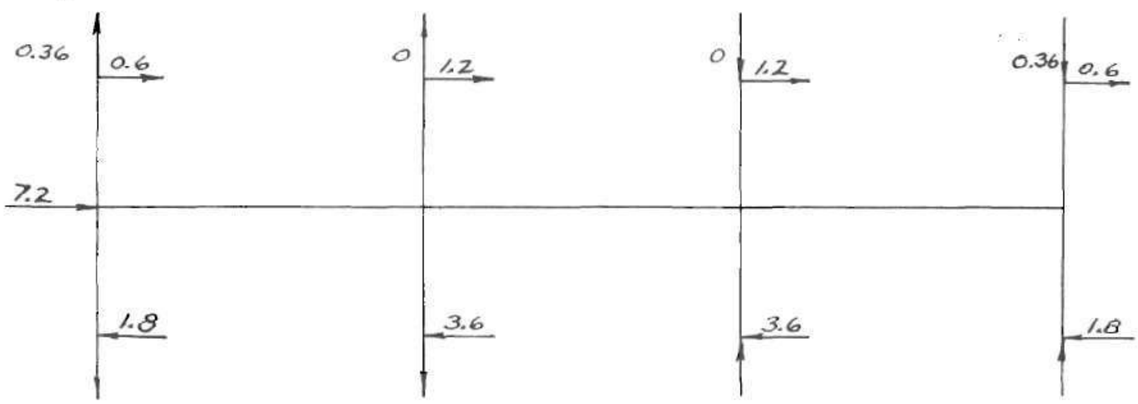
$$3.00 - 1.20 - F = 0$$

$$F = 1.80 \text{ kips}$$

for the compression in the girder.

Since the bent is symmetrical about the center line, the moments and shears of the members of the right half will be the same as those of the left half and therefore will not be computed again.

Considering that part of the structure between the points of contraflexure of the tenth and ninth stories we have the independent structure shown below.



Each aisle of the ninth story must take one-third of the total horizontal forces acting on the structure above the ninth story, which is 3.60 kips per aisle. Therefore, the horizontal shear in each interior column will be 3.60 kips while the horizontal shear in each exterior column will be 1.80 kips.

The moments and shears of the various members are calculated in the same manner as for the tenth story.

V=0.36 M=3.60 H=0.60 M=3.60 T=0.36	V=0.36 M=3.60 H=1.20 M=7.20 T=0.00	
V=1.44 M=14.4 H=1.80 M=10.8 T=1.80	V=1.44 M=14.4 H=3.60 M=21.6	
V=2.88 M=28.8 H=3.00 M=18.0 T=4.68	V=2.88 M=28.8 H=6.00 M=36.0	
V=4.32 M=43.2 H=4.20 M=25.2 T=9.00	V=4.32 M=43.2 H=8.40 M=50.4	
v=5.76 M=57.6 H=5.40 M=32.4 T=14.76	V=5.76 M=57.6 H=10.8 M=64.8	
V=7.20 M=72.0 H=6.60 M=39.6 T=21.96	V=7.20 M=72.0 H=13.2 M=79.2	
V=8.64 M=86.4 H=7.80 M=46.8 T=30.6	V=8.64 M=86.4 H=15.6 M=93.6	
V=10.08 M=100.8 H=9.00 M=54.0 T=40.68	V=10.08 M=100.8 H=18.0 M=108.0	
V=11.52 M=115.2 H=10.2 M=61.2 T=52.2	V=11.52 M=115.2 H=20.4 M=122.4	
V=15.4 M=154.0 H=11.6 M=92.8 T=67.6	V=15.4 M=154.0 H=23.2 M=185.6	

THE PORTAL METHOD

THE METHOD OF SUTHERLAND AND BOWMAN

This method is advocated by Sutherland and Bowman in their book "Structual Theory". It is based upon the following assumptions:

1. The points of contraflexure in exterior girders are located at 0.55 of their length from their outer ends and in other girders at their mid-points except (a) in the center aisle where the total number of aisles is odd and (b) in the two aisles nearest the center where the total number of aisles is even. In these excepted cases the points of contraflexure in girders will be located as required by the conditions of symmetry and equilibrium.
2. In bents of one or more stories, the points of contraflexure in bottom story columns are at 0.60 height from the base; in bents of two or more stories, the points of contraflexure in top story columns are at 0.65 height from the top; in bents of three or more stories, the points of contraflexure in the columns of the story next to the top are at o.60 height from the upper end; in bents of four or more stories, the points of contraflexure in the columns of the second story from the top are at 0.55 height from the upper end; and in bents of five or more stories the points of contraflexure in the stories not provided for above are at mid-height.

3. There is divided equally among the columns of the bottom story an amount of shear equal to

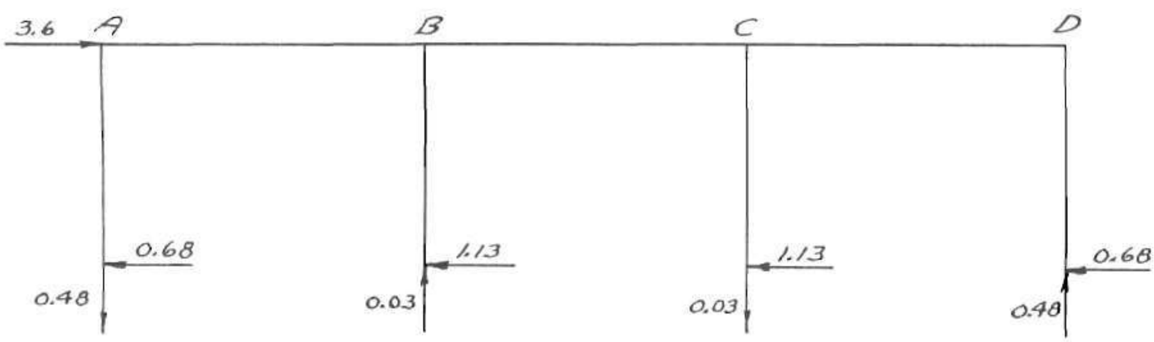
$$\frac{\text{number of aisles} - \frac{1}{2}}{\text{number of columns}}$$

times the total shear in the story, the remaining shear in the bottom story is divided among the aisles inversely as their widths, and the shear in an aisle is divided equally between the two columns adjacent to the aisle. There is divided equally among the columns of the other stories an amount of shear equal to

$$\frac{\text{number of aisles} - 2}{\text{number of columns}}$$

times the total shear in the story. The remaining shear in the story is divided among the aisles inversely as their widths, and the shear in an aisle is divided equally between the two columns adjacent to the aisle.

Considering that part of the structure above the points of contraflexure of the tenth story columns we have the independent structure as shown below.



Determining the shear in the columns we have

$$\frac{3 - 2}{4} \times 3.60 = 0.90 \text{ kip}$$

to be divided equally among the four columns, which gives

$$\frac{0.90}{4} = 0.23 \text{ kip per column}$$

The remaining shear is equal to

$$3.60 - 0.90 = 2.70 \text{ kips}$$

and is divided equally among the three aisles, which gives

$$\frac{2.70}{3} = 0.90 \text{ kip per aisle}$$

Each exterior column takes one-half the shear of an aisle, or 0.45 kip. Each interior column takes the total shear of an aisle, 0.90 kip.

The total shear taken by an exterior column is

$$0.45 + 0.23 = 0.68 \text{ kip.}$$

The total shear taken by an interior column is

$$0.90 + 0.23 = 1.13 \text{ kip.}$$

The points of contraflexure of the tenth story columns are 0.65 x 12 = 7.8 feet down from the top.

The moment in the upper part of an exterior column is

$$M = 7.8 \times 0.68 = 5.30 \text{ kip-ft.}$$

The moment in the lower part is

$$M = 4.2 \times 0.68 = 2.86 \text{ kip-ft.}$$

The moment in the upper part of an interior column is

$$M = 7.8 \times 1.13 = 8.81 \text{ kip-ft.}$$

The moment in the lower part of an interior column is

$$M = 4.2 \times 1.13 = 4.75 \text{ kip-ft.}$$

The points of contraflexure of the exterior girders are 0.55 x 20 = 11 feet from the outer ends. The point of contraflexure of the interior girder is at the center. The moment in the outer part of an exterior girder must balance the moments in the exterior columns at that joint. Hence, the moment in the outer part of an exterior girder of the tenth story is 5.30 kip-ft. The shear in the exterior girder is

$$\frac{5.30}{11} = 0.48 \text{ kip}$$

The moment in the inner part of an exterior girder is

$$M = 0.48 \times 9 = 4.32 \text{ kip-ft.}$$

The moment in the interior girder is

$$8.81 - 4.32 = 4.49 \text{ kip-ft.}$$

The shear in the interior girder is

$$\frac{4.49}{10} = 0.45 \text{ kip}$$

The direct stress in a column is found by taking a summation of the vertical forces which act within the points of contraflexure, therefore the direct stress in an exterior column of the tenth story is equal to the shear in the exterior girder which is 0.48 kip.

The moments, shears and stresses of the other members of the structure are determined in the same manner.

H=0.68 M=5.26 above M=2.84 below T=0.48	V=0.45 M=4.48 H=1.13 M=8.78 above M=4.73 below C=0.03	V=0.48 M=5.26 right M=4.32 left
H=2.03 M=14.62 M=9.74 T=0.96	V=1.47 M=14.68 H=3.37 M=24.26 M=16.18 C=0.15	V=1.59 M=17.46 M=14.31
H=3.38 M=22.3 M=18.25 T=2.55	V=2.71 M=27.08 H=5.62 M=37.1 M=30.35 C=0.35	V=2.91 M=32.04 M=26.20
H=4.73 M=28.4 T=5.46	V=3.94 M=39.37 H=7.87 M=47.2 C=0.65	V=4.24 M=46.65 M=38.18
H=6.08 M=36.5 T=9.70	V=5.48 M=54.8 H=10.12 M=60.60 C=0.86	V=5.89 M=64.9 M=53.0
H=7.43 M=44.55 T=15.59	V=6.85 M=68.45 H=12.37 M=74.15 C=1.37	V=7.36 M=81.05 M=66.30
H=8.78 M=52.6 T=22.95	V=8.25 M=82.5 H=14.62 M=87.80 C=1.95	V=8.83 M=97.15 M=79.45
H=10.13 M=60.8 T=31.78	V=9.63 M=96.3 H=16.87 M=101.2 C=2.62	V=10.3 M=113.4 M=92.70
H=11.48 M=68.8 T=42.08	V=11.01 M=110.1 H=19.12 M=114.0 C=3.39	V=11.78 M=129.6 M=106.0
H=15.27 M=97.60 above M=156.5 below T=57.22	V=10.41 M=104.1 H=19.58 M=125.2 above M=178.0 below C=8.12	V=15.14 M=166.4 M=136.0

THE MRTHOD OF SUTHERLAND & BOWMAN

UNIT OR WORKING STRESSES

Since wind loads are intermittent and seldom reach their maximum, greater working stresses are permissible for them than for live and dead loads. All engineers recognize this fact and it has found a place in most building codes. In the New York City code a working stress of 16,000 pounds per square inch is specified for tension in rolled steel, an excess of 50 per cent of stresses prescribed elsewhere in the code is allowed for combined wind, dead and live loads, provided that the sections thus found are not less than those required by the dead and live loads alone. In Chicago where 18,000 pounds per square inch is the basic unit stress for tension an excess of thirty-three per cent is allowed for combined stresses, thus permitting in both New York and Chicago a working stress of 24,000 pounds per square inch tension for combined loads. The National Board of Fire Underwriters recommends this same unit stress.

CONCLUSION

Each of the foregoing approximate methods has its advocates. At the present time the Cantilever Method and the Portal Method are probably used more than any of the others. The Cantilever Method was used by Mr. Robins Fleming, structural engineer for the American Bridge Co., in designing the twenty story Finance Building in Philadelphia. He used the Portal Method in designing the eighteen story Hurt Building in Atlanta. Of recent buildings the Portal Method was used by Hurlbut and Van Vleck in the wind stresses in the Lincoln Building, New York City, seven hundred feet high with offices on the fifty-third floor. The Cantilever Method was used in designing the American Insurance Union Building, Columbus and the four hundred and fifty foot Foshay Tower, Minneapolis, thirty-three stories, eighty-nine by eighty-seven feet at the base with sides sloping like the Washington Monument. The Cantilever Method is followed by the Bethlehem Steel Company Handbook in its formula for determining wind stresses and J.E.Kirkham advocates it in his "Structual Engineering". Professor Ketchum in his "Steel Mill Buildings" uses the Portal Method. As the two methods do not give the same results for the stress in a member it may be inferred that one of them must be in error, but Professor Burr says, "So long as the stresses found by one legitimate method of analysis are provided for, the stability of the structure is assured".

The other methods sometimes used to calculate wind stresses are to a certain extent merely variations from the Cantilever or Portal Methods. It can be seen that the shears in the girders are the same when calculated by the Method of Equal Shears as when calculated by the Portal Method; and, the Continuous Portal Method gives the same girder shear as the Cantilever Method. The shears in the columns are the same when determined by the Method of Equal Shears as when determined by the Continuous Portal Method. The results obtained by the Method of Sutherland and Bowman differ altogether from those of the other methods since the points of contraflexure of the members are not considered to be at the center of the member. The authors of this method beleive it to be the most accurate of the approximate methods. The Continuous Portal Method can not be applied to traverse bents of more than four aisles. In bents of five aisles or more the points of contraflexure of girders in the leeward aisle fall outside of the girders and the method fails. Table II shows a comparison of the stresses in the members of the fifth story as determined by the various approximate methods under discussion.

TABLE II.

STRESSES IN MEMBERS OF THE FIFTH STORY

shear in kips
moments in kip-ft.

Method	C	P	E.S.	C.P.	S.&B.
H ext. col.	5.94	6.60	9.90	9.90	7.43
H int. col.	13.86	13.20	9.90	9.90	12.37
V ext. gir.	6.48	7.20	7.20	6.48	7.36
V int. gir.	8.64	7.20	7.20	8.64	6.85
M ext. col.	35.64	39.60	59.40	59.40	44.55
M int. col.	83.16	79.20	59.40	59.40	74.15
M ext. gir.	64.80	72.00	72.00	64.80	81.05
M int. gir.	86.40	72.00	72.00	86.40	68.45

BIBLEOGRAPHY

"Continuous Frames of Reinforced Concrete" - Cross and Morgan

"Wind Stresses in Buildings" - Robins Fleming

"Stresses in Framed Structures" - Hool and Kinne

"Structural Theory" - Sutherland and Bowman

"Framed Structures and Girders" - Edgar Marburg

"Practical Design of Wind Bracing" - Clyde T. Morris

"Structural Engineering"- J. E. Kirkham